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Constant Volume Limit of Pulsed Propulsion for a Constant γ Ideal Gas

Douglas G. Talley* and Edward B. Coy†
U.S. Air Force Research Laboratory, Edwards Air Force Base, California 93524-7660

The constant volume (CV) limit of pulsed propulsion is explored theoretically, where the combustion chamber is approximated as being time varying but spatially uniform, and the nozzle flow is approximated as being one dimensional but quasi steady. Isentropic blowdown of a constant γ ideal gas is assumed. Notably, all of the fixed expansion ratio results can be expressed as analytical solutions. Using an appropriately selected fixed expansion ratio nozzle was found not to result in more than a 3% performance penalty over using a variable expansion ratio nozzle optimized at all times for the instantaneous ratio of chamber to ambient pressure. It was also found that an optimized CV device produces more impulse than an optimized constant pressure device operating at the fill pressure of the CV device, for all ambient pressures except in a vacuum. However, the latter conclusion applies to optimum expansion ratios, which are infinite in a vacuum. Caution is required in applying the latter conclusion to the finite expansion ratios of real devices.

Nomenclature

A	=	area
c	=	speed of sound
c_F	=	thrust coefficient
c_p	=	specific heat at constant pressure
F	=	thrust
$g(\gamma)$	=	Eq. (4)
I	=	impulse
M	=	Mach number
$\dot{m}$	=	rate of mass flow
P	=	pressure
R	=	gas constant
r	=	ρ/ρ_0
T	=	temperature
t	=	time
V	=	volume
v	=	velocity
γ	=	ratio of specific heats
ε	=	expansion ratio
ι	=	dimensionless impulse, $I/\rho_0 c_0 V$
ρ	=	density
τ	=	dimensionless time, $t/c_0 A^* V$
ϕ	=	P/P_∞
Ω, Φ	=	Eq. (23)

Subscripts

cp	=	pertaining to a constant pressure cycle
e	=	nozzle exit
f	=	fill condition
lc	=	pertaining to limit-cycle operation
0	=	initial condition before start of blow down
∞	=	ambient condition

Superscript

*	=	throat condition

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*Combustion Group Leader, Aerophysics Branch.
†Research Scientist, Aerophysics Branch.

Introduction

RECENT interest in pulsed detonation propulsion has spawned a number of attempts to model the system performance, the specific impulse predictions from which have tended to vary widely.[1] One limiting case from which system performance estimates can be made is the so-called constant volume (CV) limit. Pulsed detonation devices approach the CV limit because large detonation wave speeds allow combustion to be complete before the gases in the combustion chamber have time to expand appreciably beyond the original volume of the chamber, resulting in a nearly constant volume combustion process. The CV limit is approached more exactly in real devices in the limit of large combustion chambers and small nozzles, where there is ample time for transient waves to decay in the combustion chamber before an appreciable amount of mass can exit the nozzle. In this latter limit, that is, where characteristic wave transit times are much shorter than characteristic blowdown times, the CV limit can be approached even when the mechanism of combustion is not detonations and is, therefore, useful to consider as a reference case. Hence, the CV limit is referred to here simply as the CV limit of pulsed propulsion, regardless of the mechanism of combustion. When heat losses are neglected and isentropic expansions following heat addition are assumed, an upper bound for the impulse produced by a CV limit device can be calculated. In what follows, the isentropic blowdown of a constant γ ideal gas in a CV limit device is explored, where the combustion chamber is approximated as being spatially uniform but time varying, while the flow in the nozzle is approximated as being quasi steady and one dimensional. The results are compared to the performance of conventional constant pressure (CP) devices.

The CV limit as envisioned herein corresponds to the conceptual device shown in Fig. 1. An instantaneously actuated propellant valve controls the introduction of fresh reactants, while an instantaneously actuated valve in the nozzle controls the exhaust of combustion products. Time $t = 0^-$ corresponds to an initial condition with both valves closed, where the combustion chamber is initially filled with fresh reactants at some uniform fill pressure P_f, uniform fill temperature T_f, and uniform fill density ρ_f. Heat is then added, and both valves remain closed until all transients decay, at which time the chamber will have relaxed to some initial uniform combustion pressure P_0, initial uniform combustion temperature T_0, and initial combustion density $\rho_0 = \rho_f$. The pressure rise P_0/P_f is not specified and is left as a free parameter. For reactants initially

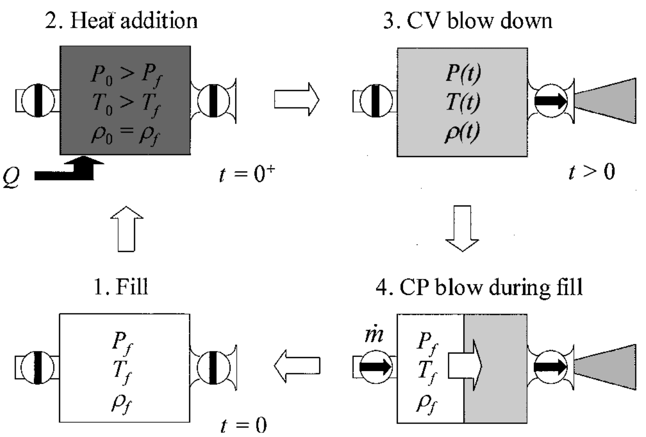

Fig. 1 CV limit cycle.

in the gaseous state, the pressure rise P_0/P_f may be on the order of 10. For reactants initially in a liquid state, the pressure rise may be more on the order of 1000. Heat addition and the decay of transients is assumed to take zero time and to be complete by time $t = 0$. The nozzle valve then opens instantaneously at time $t = 0$, CV blowdown begins, impulse begins to be produced, and the pressure, temperature, and density become functions of time, although they remain spatially uniform. The combustion products are assumed to be calorically perfect, with constant specific heats and molecular weight throughout the blowdown. Heat losses from the chamber are neglected, and the blowdown is considered to be isentropic throughout the chamber and nozzle. The flow in the nozzle is assumed to vary one dimensionally, but is quasi steady at any instant of time. Blowdown occurs at least until the chamber pressure reaches the fill pressure P_f. The propellant valve is then instantaneously opened, introducing fresh reactants, which push the remaining products out. The blowdown changes at this point from having a constant volume and varying pressure in the chamber to having a constant pressure and varying volume (of the products) in the chamber. The fill of fresh reactants continues until the chamber is full, at which time both valves close instantaneously, and the system is returned to its initial state.

A mathematical model of the CV limit under these assumptions is formulated next.

Formulation

General

The impulse produced by the unsteady blowdown of the combustion gases in the CV limit is the integral of the thrust $F = \dot{m} v_e + (P_e - P_\infty) A_e$ over time, where P_∞ is the ambient pressure and v_e, P_e, and A_e are the velocity, pressure, and area, respectively, at the nozzle exit plane. Under the transformation $\mathrm{d}t = (\mathrm{d}t/\mathrm{d}\rho)\,\mathrm{d}\rho = -(V/\dot{m})\,\mathrm{d}\rho$ and the further transformation $r \equiv \rho/\rho_0$, where V is the combustion chamber volume and ρ_0 is the initial chamber density after combustion but before blowdown at time $t = 0$, the total impulse I and blowdown time t may be expressed as

$$I(t) = \int_0^t F(t)\,\mathrm{d}t = \rho_0 V \int_r^1 \left[v_e + \frac{(P_e - P_\infty) A_e}{\dot{m}} \right] \mathrm{d}r \quad (1)$$

$$t = \int_0^t \mathrm{d}t = \rho_0 V \int_r^1 \frac{\mathrm{d}r}{\dot{m}} \quad (2)$$

Fixed Nozzles

For quasi-steady isentropic flow, the instantaneous exit velocity is $v_e = \sqrt{[2c_p(T - T_e)]}$, where T and T_e are the instantaneous chamber and exit temperatures, respectively. Pressures and densities will be related in a simple fashion to temperatures according to $T/T_0 = (\rho/\rho_0)^{\gamma - 1} = (P/P_0)^{(\gamma - 1)/\gamma}$, where γ is the ratio of specific heats and where the subscript 0 denotes conditions in the chamber at time $t = 0$. With $c_0 = \sqrt{(\gamma R T_0)}$ and $c_p = \gamma R/(\gamma - 1)$, the exit velocity can be expressed as

$$v_e = c_0 r^{(\gamma - 1)/2} [2/(\gamma - 1)]^{\frac{1}{2}} \sqrt{1 - r_e^{\gamma - 1}} \quad (3)$$

where $r_e \equiv \rho_e/\rho$ is the ratio of the density at the exit plane to the density in the chamber. For isentropic nozzle flow, the exit density ratio is related to the expansion ratio $\varepsilon \equiv A_e/A^*$, where A^* is the throat area, by (see Eq. 5.3 in Ref. 2)

$$\varepsilon = \frac{g(\gamma)}{r_e \sqrt{1 - r_e^{\gamma - 1}}}$$

$$g(\gamma) \equiv \left(\frac{\gamma - 1}{2}\right)^{\frac{1}{2}} \left(\frac{2}{\gamma + 1}\right)^{(\frac{1}{2})(\gamma + 1)/(\gamma - 1)} \quad (4)$$

Equation (4) has two solutions for r_e corresponding to subsonic and supersonic flow at the exit plane. These solutions will depend only on ε and γ. For supersonic flow, the density ratio r_e will therefore remain constant during blowdown as long as compression waves do not enter the nozzle, and Eq. (3) will be analytically integrable when inserted into Eq. (1). Compression/expansion waves will remain external to the nozzle for all r such that

$$r > (1/r_e)(\phi_e'/\phi_0)^{1/\gamma} \quad (5)$$

where $\phi_0 \equiv P_0/P_\infty$ and $\phi_e \equiv P_e/P_\infty$ are the ratios of the initial chamber and nozzle exit pressures, respectively, to the ambient pressure, and where ϕ_e' is the critical value of ϕ_e below which a compression wave enters the nozzle. The critical value of ϕ_e is given by (see Eqs. 2.48a and 5.2 of Ref. 2)

$$\phi_e' = \left\{1 + [2\gamma/(\gamma + 1)]\left(M_e^2 - 1\right)\right\}^{-1} \quad (6)$$

where

$$M_e^2 = [2/(\gamma - 1)]\left[(1/r_e)^{\gamma - 1} - 1\right] \quad (7)$$

While Eq. (5) holds, the flow will be choked, and the throat conditions will be functions only of γ. For a throat density and velocity given by (see Eq. 2.35a of Ref. 2) $\rho^* = r\rho_0[2/(\gamma + 1)]^{1/(\gamma - 1)}$ and $v^* = \sqrt{(\gamma R T^*)} = c_0[2/(\gamma + 1)]^{1/2} r^{\gamma - 1}$, and with $P_e = P_0 r_e^\gamma r^\gamma$, the instantaneous mass flow and thrust become

$$\dot{m}(r) = \rho^* A^* v^* = \rho_0 c_0 A^* g(\gamma) [2/(\gamma - 1)]^{\frac{1}{2}} r^{(\gamma + 1)/2} \quad (8)$$

$$F(r)/P_0 A^* g(\gamma) = r^\gamma \gamma [2/(\gamma - 1)] \sqrt{1 - r_e^{\gamma - 1}} + \left(1/r_e\sqrt{1 - r_e^{\gamma - 1}}\right)\left(r_e^\gamma r^\gamma - P_\infty/P_0\right) \quad (9)$$

where Eq. (4) has also been used. Thus all terms in Eq. (1) will integrable analytically.

Define $\iota \equiv I/\rho_0 c_0 V$ and $\tau \equiv t c_0 A^*/V$ to be a dimensionless impulse and a dimensionless time, respectively. Then integrating Eqs. (1) and (2) gives

$$\iota(r) = \sqrt{1 - r_e^{\gamma - 1}} \left(\frac{2}{\gamma - 1}\right)^{\frac{1}{2}} \left(\frac{2}{\gamma + 1}\right) a(r) + \frac{1}{\gamma r_e \sqrt{1 - r_e^{\gamma - 1}}} \left(\frac{\gamma - 1}{2}\right)^{1/2} \times \left[r_e^\gamma \frac{2}{\gamma + 1} a(r) - \frac{1}{\phi_0}\left(\frac{2}{\gamma - 1}\right) b(r) \right] \quad (10)$$

$$\tau(r) = \frac{1}{g(\gamma)} \left(\frac{2}{\gamma - 1}\right)^{\frac{1}{2}} b(r) \quad (11)$$

where

$$a(r) \equiv 1 - r^{(\gamma+1)/2} \qquad b(r) \equiv (1/r)^{(\gamma-1)/2} - 1 \tag{12}$$

The quantity ι/τ will be related to the average thrust $\bar{F} = I/t$. Noting that $c_0^2 = \gamma R T_0$ and $P_0 = \rho_0 R T_0$, then combining constants after dividing, leads to

$$\gamma \iota/\tau = \bar{F}\big/P_0 A^* \equiv \bar{c}_F \tag{13}$$

where $\bar{c}_F$ is the average thrust coefficient for the blowdown.

Variable Nozzles

Fixed expansion ratio nozzles will be optimized for at most only a single value of r during the blowdown; losses will occur at all other values. To assess what the performance would be if these losses were not present, a variable nozzle can be envisioned where the expansion ratio is continuously adjusted to maintain the optimum expansion ratio for the given ratio of chamber to ambient pressure at all times. The variable nozzle limit produces the maximum possible impulse and is, therefore, useful as a reference case.

With the exit pressure always matched to the ambient pressure, only the exit velocity term $v_e = \sqrt{[2c_p(T - T_e)]}$ in Eq. (1) will be of concern. For fixed nozzles, it was found that the term $T_e/T = r_e^{\gamma-1}$ was a constant, but here the exit temperature is fixed by the exit conditions $P_e = P_\infty$, $\rho_e = \rho_0(1/\phi_0)^{1/\gamma}$, and $T_e = T_0(1/\phi_0)^{(\gamma-1)/\gamma}$. Therefore T_e/T will not be constant in this case. The dimensionless impulse in this case becomes

$$\iota(r) = \left(\frac{2}{\gamma - 1}\right)^{\frac{1}{2}} \int_r^1 \sqrt{r^{(\gamma-1)} - \left(\frac{1}{\phi_0}\right)^{(\gamma-1)/\gamma}}\, \mathrm{d}r \tag{14}$$

Equation (14) cannot be integrated analytically but may easily be integrated numerically.

The blowdown time for the variable nozzle limit will depend on the mass flow, and calculation of the mass flow will depend on whether the exit area or the throat area is varied to optimize the expansion ratio. The mass flow is best calculated using whichever area is being held constant. If the exit area is assumed to be varied, as might approximately be the case with an automatically compensating nozzle such as a plug nozzle, the mass flow and blowdown time based on a constant throat area are still given by Eqs. (8) and (11), respectively. If the throat area is varied, then the mass flow and blowdown time for a fixed exit area are

$$\dot{m} = \rho_e A_e v_e = \rho_0 c_0 A_e \left(\frac{2}{\gamma - 1}\right)^{\frac{1}{2}} \left(\frac{1}{\phi_0}\right)^{1/\gamma} \sqrt{r^{(\gamma-1)} - \left(\frac{1}{\phi_0}\right)^{(\gamma-1)/\gamma}} \tag{15}$$

$$\tau_e(r) = \phi_0^{1/\gamma} \left(\frac{\gamma - 1}{2}\right)^{\frac{1}{2}} \int_r^1 \frac{\mathrm{d}r}{\sqrt{r^{(\gamma-1)} - (1/\phi_0)^{(\gamma-1)/\gamma}}} \tag{16}$$

where $\tau_e = t c_0 A_e / V$ is defined this time in terms of the exit area. Equation (16) also cannot be integrated analytically but may easily be integrated numerically. Inasmuch as shocks and expansion waves will not occur when the expansion ratio is always matched to the instantaneous pressure ratio, Eqs. (14–16) for the variable nozzle case will apply until the expansion ratio reduces to unity and the flow becomes unchoked, that is, for all r such that

$$r > [(\gamma + 1)/2]^{1/(\gamma-1)} (1/\phi_0)^{1/\gamma} \tag{17}$$

Limit-Cycle Operation

In repetitive operation, intake valves are opened at some point during the blowdown to introduce fresh propellants for the next cycle, as shown in Fig. 1. Any remaining combustion products not yet expelled would be pushed out, or purged, by the incoming fresh propellants. If the purge of the remaining combustion products can be approximated as a CP process, and the value of r is large enough to prevent shocks from entering the nozzle, then the preceding equations can be used to calculate the time to purge the remaining mass and the additional impulse produced. Specifically, the time required to purge the remaining mass is $t_{\mathrm{cp}}(r) = \rho V/\dot{m} = \rho_0 V r/\dot{m}(r)$, and the additional impulse produced is $I_{\mathrm{cp}}(r) = F(r) t_{\mathrm{cp}}(r)$, where $F(r)$ and $\dot{m}(r)$ are given by Eqs. (8) and (9) at a constant value of r during the purge. The additional dimensionless impulse and blowdown times can still be expressed in the forms of Eqs. (10) and (11), but with the functions $a(r)$ and $b(r)$ given instead by

$$a_{\mathrm{cp}}(r) = [(\gamma + 1)/2]\, r^{(\gamma+1)/2} \qquad b_{\mathrm{cp}}(r) = [(\gamma - 1)/2]\, r^{(\gamma-1)/2} \tag{18}$$

The total impulse for the entire cycle can then be expressed, again in the forms of Eqs. (10) and (11), but this time with the functions $a(r)$ and $b(r)$ given by

$$a_{\mathrm{lc}}(r) = a(r) + a_{\mathrm{cp}}(r) = 1 + [(\gamma - 1)/2]\, r^{(\gamma+1)/2}$$

$$b_{\mathrm{lc}}(r) = b(r) + b_{\mathrm{cp}}(r) = [(\gamma + 1)/2](1/r)^{(\gamma-1)/2} - 1 \tag{19}$$

For limit-cycle operation with variable nozzles, the total cycle impulse is given instead by the sum of the CP impulse with the impulse calculated by Eq. (14), and the total cycle blowdown time is the sum of the CP time plus the blowdown time calculated by Eq. (11) or Eq. (16). The expansion ratio always matches the pressure ratio in the variable nozzle case, so that the exit density ratio must be set to $r_e = (1/r)(1/\phi_0)^{1/\gamma}$.

The density ratio r cannot be specified arbitrarily if limit-cycle operation is to be achieved. Limit-cycle operation requires the blowdown to proceed at least until the chamber pressure reaches the fill pressure at which fresh propellants are introduced. This gives $r \leq (P_f/P_0)^{1/\gamma}$, where P_f is the fill pressure. The inequality sign indicates that the blowdown can proceed to a pressure lower than the fill pressure if the fresh reactants undergo compression during the fill. Given that $\rho_0 = \rho_f$ for CV heat addition, the ratio P_f/P_0 could be on the order of 1000 times smaller for propellants in an initial liquid state compared to an initial gaseous state.

For a given density ratio r, the blowdown of the remaining combustion products in a CP mode produces a greater impulse than if the blowdown had been permitted to proceed to a lower density ratio. Therefore, given a choice, the density ratio r should be selected to be the maximum value possible consistent with limit-cycle operation and available injection pressure, namely, $r = (P_f/P_0)^{1/\gamma}$.

Comparisons with CP Devices

Comparisons of the dimensionless impulse or the thrust coefficient with corresponding quantities for CP processes can be performed if the normalization constants are the same in both cases. Here the normalization constants are chosen to be those of the CV process, leading to the correction factors Ω and Φ in the subsequent resulting expressions. When it is assumed that the expansion ratio for the CP process is the optimum expansion ratio, the resulting dimensionless impulse, blowdown time, and average thrust coefficient become

$$\iota_{\mathrm{cp}} = (1/\Omega)[2/(\gamma - 1)]^{\frac{1}{2}} \sqrt{1 - (\Phi/\phi_0)^{(\gamma-1)/\gamma}} \tag{20}$$

$$\tau_{\mathrm{cp}} = (\Phi/\Omega)[1/g(\gamma)][(\gamma - 1)/2]^{\frac{1}{2}} \tag{21}$$

$$\bar{c}_{F,\mathrm{cp}} = \gamma g(\gamma)[2/(\gamma - 1)](1/\Phi)\sqrt{1 - (\Phi/\phi_0)^{(\gamma-1)/\gamma}} \tag{22}$$

where

$$\Omega \equiv c_0/c_{\mathrm{cp}} \qquad \Phi \equiv P_0/P_{\mathrm{cp}} \tag{23}$$

and where $\iota_{\mathrm{cp}} \equiv I_{\mathrm{cp}}/\rho_0 c_0 V$, $\tau_{\mathrm{cp}} \equiv t_{\mathrm{cp}} c_0 A^*/V$, $\bar{c}_{F,\mathrm{cp}} \equiv \bar{F}/P_0 A^*$, and the values of ρ_0, c_0, P_0, and V are those of the CV process. The quantity τ_{cp} in this context can be interpreted to be the dimensionless time required by the CP process to expel the same mass as the initial mass of the CV process.

Results and Discussion

Effect of Thermochemistry

The dimensionless impulse and blowdown time, Eqs. (10–12), depend explicitly on r and implicitly only on γ, r_e (or ε), and ϕ_0. Of these, only γ depends on the thermochemistry, but this dependency is weak. The major influence of thermochemistry comes through the initial speed of sound c_0 used to normalize the impulse. Thus, the specific impulse $I_s = \iota c_0$ is maximized when c_0 is maximized. This in turn implies that the optimum thermochemistry is that which maximizes the initial combustion temperature T_0 and minimizes the molecular weight. The same general guidance is known to also apply to CP devices. Thus CV devices should optimize at approximately the same mixture ratios as CP devices. Also, like CP devices, the thrust is maximized by maximizing the initial chamber pressure and the throat area, as implied by Eq. (9).

Fixed Nozzles

Much can be understood about the blowdown of fixed nozzles by examining conventional steady-state thrust coefficient curves such as can be found in standard texts.[3] The thrust coefficient is defined in Eq. (13), where P_0 in the steady-state case is interpreted to be the steady chamber pressure of a constant pressure device. A set of these curves for the steady-state case is given in Fig. 2 for $\gamma = 1.2$. These curves were generated by setting $r = 1$ in Eq. (18), substituting into Eqs. (10) and (11), and then calculating $\bar{c}_F$ per Eq. (13). The curves reproduce those given in Ref. 3. Dimensionless isobars (curves of constant ϕ_0) initially increase with ε, reach a maximum, then decrease to a minimum where a shock enters the nozzle and the preceding formulation is no longer valid. The curve for $\phi_0 = \infty$ reaches a maximum only at $\varepsilon = \infty$. A curve connecting the maxima indicates the expansion ratio producing the maximum thrust for a given isobar.

The blowdown of a fixed-nozzle CV device proceeds along a vertical path of constant ε between two isobars. The thrust produced will be some average between the two isobars. When any two isobars in Fig. 2 are picked, for example, between $\phi_0 = 50$ and 20, and the vertical line between them for various ε is followed, it can be envisioned that the average thrust will reach a maximum neither at large ε nor at $\varepsilon = 1$, but at some optimum ε in between. However, the average thrust cannot be calculated directly from Fig. 2, because the thrust coefficient is proportional to the thrust divided by the chamber pressure, not the thrust alone. A blowdown from $\phi_0 = 50$ to 20 is replotted in Fig. 3 where the curve for $\phi_0 = 20$ is normalized by the same pressure as for $\phi_0 = 50$, making all curves proportional to the thrust. This is accomplished by multiplying the thrust coefficient of Fig. 2. by 0.4 for $\phi_0 = 20$. The curve labeled b.d. gives the average thrust coefficient for the blowdown and is seen to be an average of the curves for $\phi_0 = 50$ and 20, as expected. The variable nozzle case shown in Fig. 3 will be discussed later.

Average thrust coefficients are plotted as functions of ϕ_0 and ε for blowdowns to $r = 0.75$, 0.5, and 0.25 in Figs. 4, 5, and 6, respectively. Dimensionless blowdown times are plotted as a function of r for two values of γ in Fig. 7. As can be seen from Eq. (11)

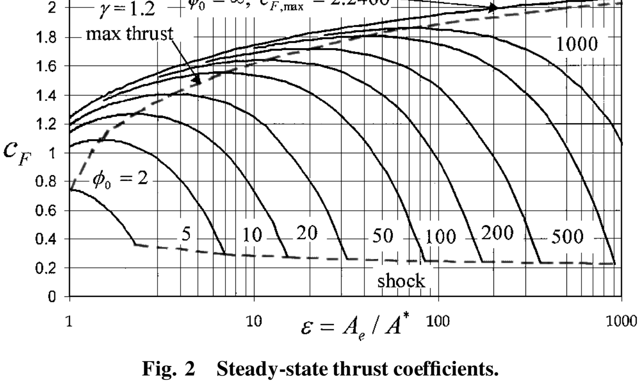

Fig. 2 Steady-state thrust coefficients.

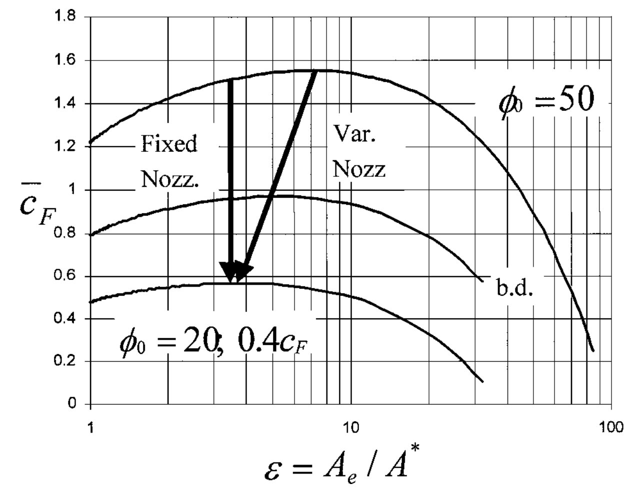

Fig. 3 Blowdown from ϕ_0 = 50 to 20, where the b.d. curve is for the blowdown; arrows indicate paths taken for fixed and variable nozzles.

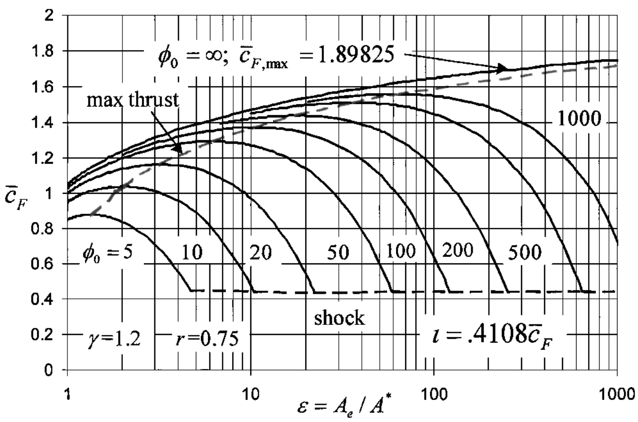

Fig. 4 Blowdown to r = 0.75.

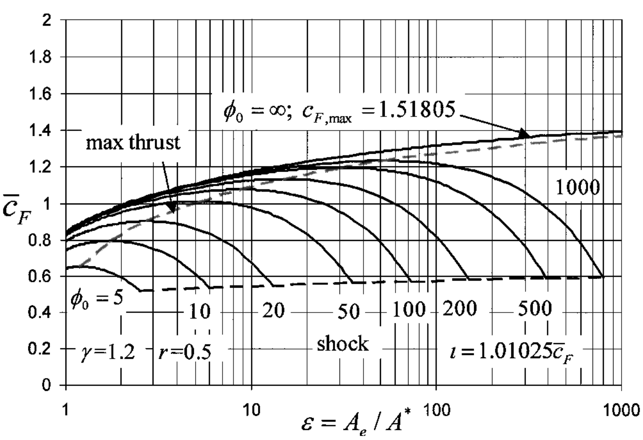

Fig. 5 Blowdown to r = 0.5.

and Fig. 7, blowdown times are independent of ϕ_0 and depend only weakly on γ. Because τ is fixed and independent of ϕ_0 for a fixed r, the curves in Figs. 4–6 will also be proportional to the dimensionless impulse ι, as can be shown from Eq. (13). However, the constant of proportionality will be different for each r, increasing as r decreases, because τ increases as r decreases. The constant of proportionality is given in Figs. 4–6. The overall trend in Figs. 4–6 is that the average thrust coefficient decreases as r decreases. This is due to τ increasing more rapidly than ι as r decreases because ι also increases as r decreases.

The expansion ratio that produces the maximum impulse can in principle be found by taking the first derivative of Eq. (9) with respect

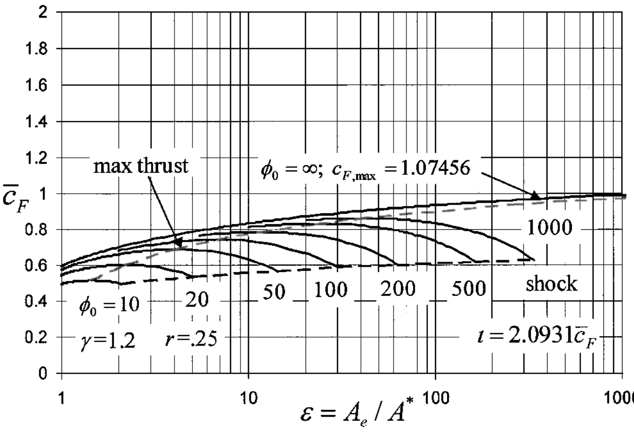

Fig. 6 Blowdown to r = 0.25.

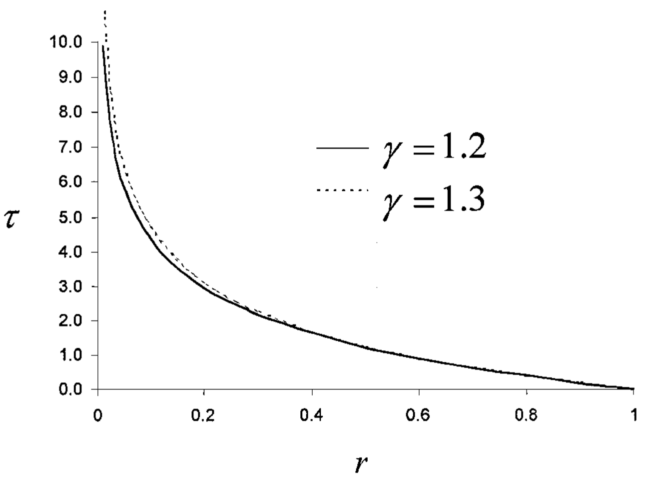

Fig. 7 Blowdown times.

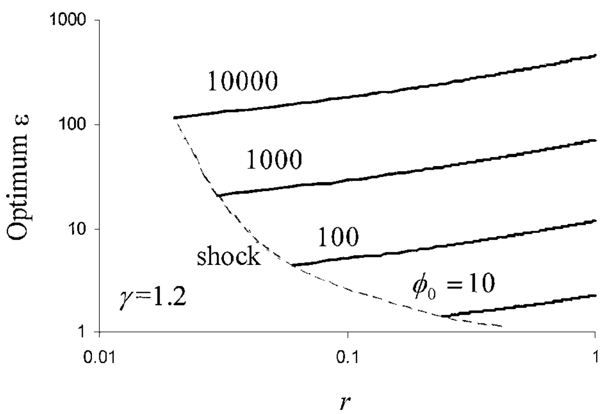

Fig. 8 Optimum expansion ratios.

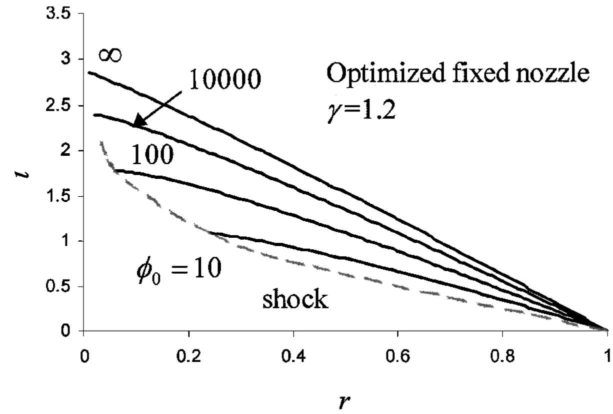

Fig. 9 Dimensionless impulse at the optimimum expansion ratio.

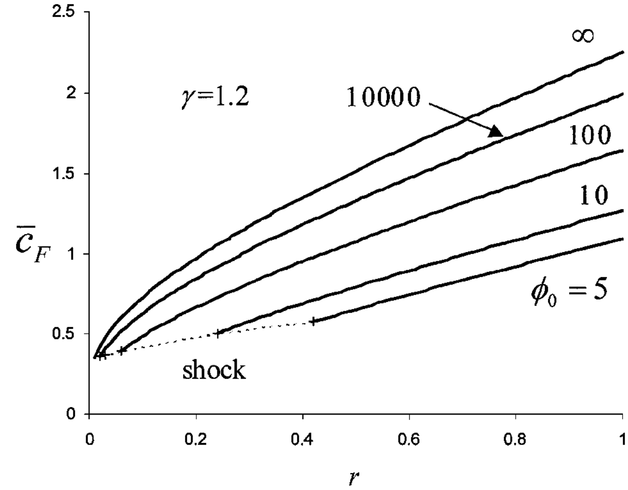

Fig. 10 Thrust coefficient at the optimum expansion ratio.

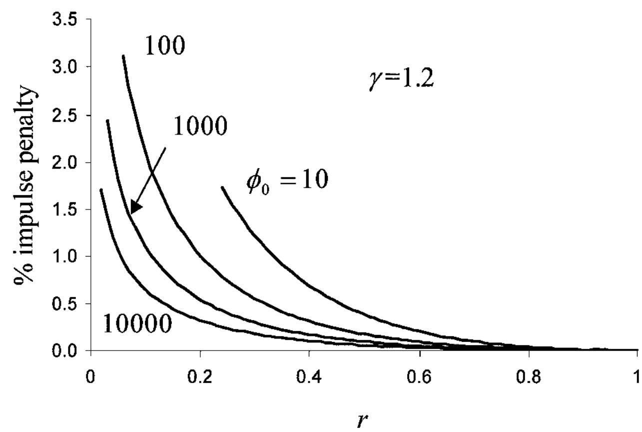

Fig. 11 Impulse penalty of an optimized fixed nozzle compared with a variable nozzle.

to r_e and setting it equal to zero. The resulting expression cannot be solved analytically, however, and so the optimum expansion ratio is computed here by finding the maximum impulse using a numerical search. The optimum expansion ratio is plotted as a function of ϕ_0 and r in Fig. 8, and the dimensionless impulse produced at the optimum expansion ratio is plotted in Fig. 9. Because r is variable, τ is also, and so the dimensionless impulse and the average thrust coefficient curves are no longer directly proportional as they were in Figs. 4–6, where r was fixed. The optimum thrust coefficient is plotted in Fig. 10.

Variable Nozzles

Because the expansion ratio is always optimized for variable nozzles, the blowdown of variable nozzles proceeds along the curve of maximum thrust in Fig. 2. The dimensionless impulse produced can be computed as a function of ϕ_0 and r by integrating Eq. (14) and compared with the performance of fixed nozzles operating at the optimum expansion ratio. The results are not easily viewed in the form of Fig. 9, however, and so they are plotted instead in Fig. 11 as the impulse penalty in using optimized fixed nozzles compared to the variable nozzle case, based on the percentage of the variable nozzle impulse. The penalty is shown not to exceed about 3% for all cases calculated. The reason the penalty is not more severe can be traced to the flatness of the curves in Fig. 2 near the points of maximum thrust. This is illustrated more clearly in Fig. 3, where the difference between the variable and optimized fixed nozzle blowdown paths is shown. As can be seen, the maximum instantaneous thrust coefficient is not that much higher for the variable nozzle case than for optimized fixed nozzles, leading to calculated average thrust coefficients that are very close.

Comparison with CP Devices

By normalization with the same constants as used for the CV device, the correction factors Ω and Φ as defined in Eq. (23) and used in Eqs. (20–22) allow the normalized quantities for CP devices to be directly compared with CV devices at any arbitrary CP chamber pressure and speed of sound (temperature). However, only certain cases will be of interest here. It is presumed that the two cycles are to be compared for the same propellant combination and at the optimum mixture ratio for each cycle, the latter of which, as discussed earlier, should be approximately the same in both cases. If T_f is the initial fill temperature before combustion and q is the heat of reaction per unit mass for a given propellant combination and mixture ratio, then the initial CP temperature after combustion will be approximately $T_{\text{cp}} = T_f + q/c_p$. If small possible differences in q between CV and CP combustion are neglected, the initial CV temperature after combustion will be approximately $T_0 = T_f + \gamma q/c_p$. Thus, Ω will vary only over a very narrow range near unity, namely, from $\Omega = 1$ for $q = 0$ to $\Omega = \gamma^{1/2}$ for $q/c_p \gg T_f$. For $\gamma = 1.2$, this corresponds to $1 \leq \Omega \leq 1.095$. The conclusion that Ω will vary only over a very narrow range near unity remains unchanged even when small possible differences in q between CV and CP combustion are considered.

The question now arises as to which of the various pressures encountered over the CV blowdown, or not encountered, should a CP device be operated at to compare with a CV device. Only two possibilities will be considered here. The first is the case where the pressure of the CP device is the same as the fill pressure of the CV device, namely, $\Phi = P_0/P_f$. This would allow a comparison of two devices having equivalent feed systems. When it is assumed that the purge pressure is the same as the feed pressure, limit-cycle operation gives $r = (P_f/P_0)^{1/\gamma}$. Given further that $P_0 = \rho_0 R T_0$ and that $\rho_0 = \rho_f$ for a CV device, where ρ_f is the density after propellant fill but before combustion, then $P_0/P_f = 10$ would be representative of propellants in an initial gaseous state, whereas $P_0/P_f = 1000$ would approach propellants in an initial liquid state. A CP device operating at the same pressure as the fill pressure of the CV device will be referred to here as a CP_f device for brevity.

The second possibility will be to compare the performance of combustion chambers experiencing the same material stresses. A very rough approximation of this would be a CP device operating at the same pressure as the peak pressure of a CV device, namely, $\Phi = 1$. A CP device operating at the same pressure as the peak pressure of the CV device will be referred to as a CP_p device, again for brevity.

The ratio of the dimensionless impulse produced by a CP_f device to the dimensionless impulse of a CV device is presented in Fig. 12, and the ratio of the dimensionless impulse produced by the CP_p device to the dimensionless impulse of a CV device is presented in Fig. 13.

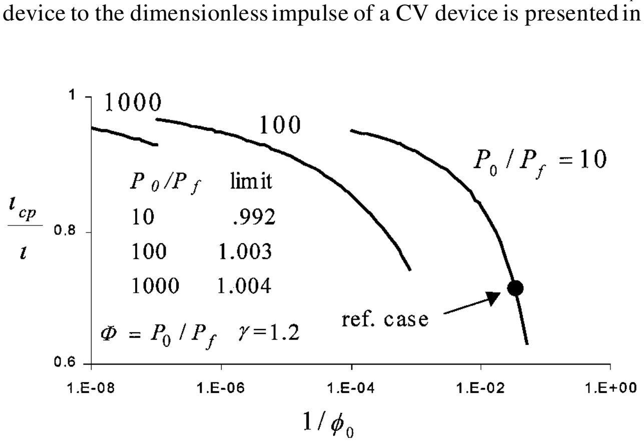

Fig. 12 Comparison of CP_f and CV impulse; reference case is stoichiometric kerosine/air at 1-atm ambient pressure and 3-atm fill pressure.

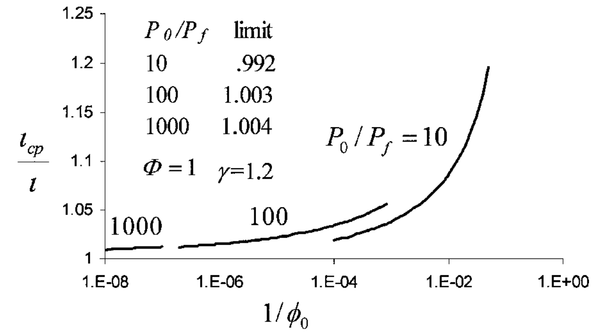

Fig. 13 Comparison of CP_p and CV impulse.

Because the same quantities are used to nondimensionalize the impulse in all cases, these plots also give the ratio of the total impulses and the ratio of the specific impulses. The expansion ratios used for the CP and CV devices are those that produce the maximum possible impulse in each case, namely, a fixed expansion ratio that is optimized for the fixed ratio of the chamber to ambient pressures for the CP device and a variable expansion ratio that changes with the variable ratio of chamber to ambient pressure for the CV device. The CV device is operated in limit-cycle mode where the blowdown is performed to the maximum value of r consistent with limit-cycle operation, $r = (P_f/P_0)^{1/\gamma}$. It is assumed that $q/c_p \gg T_f$ in all cases, giving $\Omega = \gamma^{1/2}$.

Curves are drawn in Figs. 12 and 13 for different ratios of the peak pressure to the fill pressure, P_0/P_f. Increases in this ratio indicate either more energetic chemistry (greater temperature/pressure rise) or the presence of condensed phases in the initial fill. Increases over two orders of magnitude as shown in Figs. 12 and 13 would be largely the result of the presence of condensed phases. Increases in P_0/P_f always increase the difference between the CP and CV performance. The right-hand extent of all of the curves is dictated by the shock limit, beyond which a shock enters the nozzle and the present formulation no longer applies. The left-hand extent of the curves as drawn in the Figs. 12 and 13 is limited solely for reasons of graphical clarity; the curves in both Figs. 12 and 13 actually all extend to $1/\phi_0 = 0$. The limiting values of the curves at $1/\phi_0 = 0$ are given in Figs. 12 and 13. These limiting values are exactly the same for both the CP_f and CP_p cases. The reason for this is that Eq. (20) becomes independent of the chamber pressure when $1/\phi_0 = 0$.

Away from $1/\phi_0 = 0$, the impulse of a CP_f device is shown in Fig. 12 to be inferior to that of the CV device, and the impulse of a CP_p device is shown in Fig. 13 to be superior to that of the CV device. The relative differences diminish as $1/\phi_0$ approaches zero. At large enough $1/\phi_0$, the relative differences are significantly larger than the 3% maximum performance penalty shown in Fig. 11 attributed to using an optimized fixed nozzle instead of a variable nozzle. This implies that, at large enough $1/\phi_0$, the performance penalty associated with the practical simplicity of using an optimized fixed nozzle might be tolerable compared with the complications of developing a variable nozzle. To assess whether a practically attainable device could fall within a regime where such losses might be tolerable, a reference case consisting of the approximate operating point of a stoichiometric kerosine/air (jet fuel/air) system operating at 1-atm ambient pressure and a 3-atm fill pressure is plotted as a black dot in Fig. 12. The operating point is shown to be well within a regime where such performance losses might be tolerable. Whether such losses would in fact be tolerable in a practical system would of course depend on all of the other system losses.

The case $1/\phi_0 \to 0$ is an important limit in that it represents operation where the ambient pressure approaches a vacuum, that is, in space. The trends of the curves in Figs. 12 and 13 as $1/\phi_0 \to 0$ are discussed next. Note, however, that the comparisons in Figs. 12 and 13 are for optimized expansion ratios. As $1/\phi_0 \to 0$, this means expansion ratios that approach infinity. Clearly no real device in space would operate with an infinite expansion ratio. Therefore, the curves in Figs. 12 and 13 are to be interpreted cautiously in drawing conclusions about real devices as $1/\phi_0 \to 0$. A treatment more applicable to real devices would need to account for practical considerations that constrain the finite sizes of nozzles on spacecraft. Such a treatment is beyond the scope of the present work.

For optimized expansion ratios, the difference between a CP_f device and a CP_p device vanishes as $1/\phi_0$ approaches zero, and the impulse of the CP device can become either slightly higher or slightly lower than the impulse of the CV device, depending on the magnitude of P_0/P_f. The reason for this may be found in deriving the following analytical expression for ι/ι_{cp}, which is possible when $1/\phi_0 = 0$. For limit-cycle operation, integrate Eq. (14) from $r = (P_f/P_0)^{1/\gamma}$ to $r = 1$, which now can be done analytically, and add the impulse from the CP part of the cycle by using the functions from Eq. (18) in Eq. (10). Note that $r_e = 0$ ($\varepsilon = \infty$) when $1/\phi_0 = 0$. Divide the result into Eq. (20). The result is

$$\iota_{cp}/\iota = (1/\Omega)[(\gamma + 1)/2]\left\{1 + [(\gamma - 1)/2](P_f/P_0)^{(\gamma+1)/2\gamma}\right\}^{-1} \tag{24}$$

The maximum value of the expression occurs as $P_0/P_f \to \infty$, which for $\Omega = \gamma^{1/2}$ and $\gamma = 1.2$ is $\iota/\iota_{cp} = 1.004$. As P_0/P_f becomes smaller, the magnitude of the impulse ratio also becomes smaller, until eventually it is reduced to a value less than unity. For $P_0/P_f = 10$, this value becomes $\iota/\iota_{cp} = 0.992$. Limit values of ι/ι_{cp} at $1/\phi_0 = 0$ shown in Figs. 12 and 13 were calculated using Eq. (24).

Conclusions

Analytical solutions of the CV limit of pulsed propulsion for fixed expansion ratios have been explored and compared with the case where a variable nozzle is used to match the pressure ratio at all times during the blowdown. The solutions apply for all supersonic flows where compression waves remain exterior to the nozzle. The solutions were also compared with CP devices. The findings follow.

1) CV devices should optimize at approximately the same mixture ratio as CP devices, namely, the mixture ratio that maximizes the initial temperature and minimizes the molecular weight.

2) The thrust of CV devices is maximized in the same way as for CP devices, namely, by maximizing the initial pressure and the throat area.

3) The blowdown time depends on $r = \rho/\rho_0$, only weakly depends on γ, and is independent of the initial pressure ratio $\phi_0 = P_0/P_\infty$.

4) In general, an optimum expansion ratio exists that maximizes the impulse produced by the blowdown of a CV device at a fixed expansion ratio.

5) Using an optimized fixed expansion ratio nozzle results in an impulse penalty that does not exceed 3% (for the cases considered) of the impulse that would be produced using the more complicated variable expansion ratio nozzle.

6) A comparison between a CP device operating at its optimum expansion ratio and a CV device in limit-cycle operation with a variable expansion ratio leads to the following additional conclusions.

a) Except near $1/\phi_0 = P_\infty/P_0 = 0$, the impulse produced by a CV device is superior to that of a CP device operating at the same pressure as the fill pressure of the CV device, but inferior to the impulse produced by a CP device operating at the same pressure as the peak pressure of the CV device.

b) The magnitude of the difference between a CV device and the two CP devices considered increases as $1/\phi_0$ increases.

c) At large enough $1/\phi_0$, the relative difference between a CV device and the two CP devices considered is significantly larger than the impulse penalty associated with using an optimized fixed expansion ratio instead of a variable nozzle on the CV device.

d) The regime where the relative difference between a CV device and a CP device is significantly larger than the impulse penalty associated with using an optimized fixed expansion ratio on the CV device appears to be practically achievable for at least the one reference case considered, which involved kerosine/air operation at 1-atm ambient pressure.

e) For infinite expansion ratios at $1/\phi_0 = P_\infty/P_0 = 0$, the difference between the two CP devices considered vanishes, and the impulse of the CP device can become either slightly higher or slightly lower than the impulse of the CV device, depending on the magnitude of P_0/P_f. However, these conclusions may not apply to real devices operating in space, which have finite expansion ratios.

Acknowledgments

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